Exercise 1H

1 Copy and complete the table below, simplifying expressions as shown in the first row.

	$8! - 6!$	$6!(56-1) = 39\,600$
(a)	$9! + 8!$	
(b)	$7! - 6!$	
(c)	$6! + 5!$	
(d)	$(n+1)! - n!$	
(e)	$n! - (n-1)!$	
(f)	$n! + (n-1)!$	
(g)	$(n+1)! + n!$	

(a) $9! + 8!$

$= 9 \times 8! + 8!$

$= 8!(9+1)$

$= 403\,200$

(b) $7! - 6!$

$= 7 \times 6! - 6!$

$= 6!(7-1)$

$= 4320$

(c) $6! + 5!$

$= 6 \times 5! + 5!$

$= 5!(6+1)$

$= 840$

(d) $(n+1)! - n!$

$= (n+1)n! - n!$

$= n!(n+1-1)$

$= n(n!)$

(e) $n! - (n-1)!$

$= n(n-1)! - (n-1)!$

$= (n-1)((n-1)!)$

(f) $n! + (n-1)!$

$= n(n-1)! + (n-1)!$

$= (n+1)(n-1)!$

(g) $(n+1)! + n!$

$= (n+1)n! + n!$

$= n!(n+1+1)$

$= (n+2)n!$

2 Find the value of:

a $\dfrac{8!}{4\times 6!}$ **b** $\dfrac{4!\times 5!}{3!\times 6!}$ **c** $\dfrac{10!\times 8!}{11!\times 6!}$

(a) $\dfrac{8!}{4\times 6!}$

$= \dfrac{8\times 7\times \cancel{6!}}{4\times \cancel{6!}}$

$= 14$

(b) $\dfrac{4!\times 5!}{3!\times 6!}$

$= \dfrac{4\times \cancel{3!}\times \cancel{5!}}{\cancel{3!}\times 6\times \cancel{5!}}$

$= \dfrac{2}{3}$

(c) $\dfrac{10!\times 8!}{11!\times 6!}$

$= \dfrac{\cancel{10!}\times 8\times 7\times \cancel{6!}}{11\times \cancel{10!}\times \cancel{6!}}$

$= \dfrac{56}{11}$

3 Simplify the following:

a $\dfrac{(n+1)!}{n!-(n+1)!}$ **b** $\dfrac{n!+(n+1)!}{n!}$ **c** $\dfrac{(n!)^2-1}{n!-1}$

(a) $\dfrac{(n+1)!}{n!-(n+1)!}$

$= \dfrac{(n+1)n!}{n!-(n+1)n!}$

$= \dfrac{(n+1)\cancel{n!}}{-n(\cancel{n!})}$

$= \dfrac{n+1}{-n}$

$= \dfrac{-n-1}{n}$

(b) $\dfrac{n!+(n+1)!}{n!}$

$= \dfrac{n!+(n+1)n!}{n!}$

$= \dfrac{(n+2)\cancel{n!}}{\cancel{n!}}$

$= n+2$

(c) $\dfrac{(n!)^2-1}{n!-1}$

$= \dfrac{(\cancel{n!-1})(n!+1)}{\cancel{n!-1}}$

$= n!+1$

4 Show that $\dfrac{(2n+2)!(n!)^2}{\left[(n+1)!\right]^2(2n)!} \equiv \dfrac{2(2n+1)}{n+1}$.

$\dfrac{(2n+2)!(n!)^2}{[(n+1)!]^2(2n)!}$

$= \dfrac{(2n+2)(2n+1)(\cancel{2n)!}\cancel{n!}\cancel{n!}}{(n+1)\cancel{n!}(n+1)\cancel{n!}\cancel{(2n)!}}$

$= \dfrac{2(\cancel{n+1})(2n+1)}{(n+1)(\cancel{n+1})}$

$= \dfrac{2(2n+1)}{n+1}$ shown

5 Solve for $n \in \mathbb{Z}^+$, ${}^nC_2 = 66$.

${}^nC_2 = 66$

$\frac{n!}{(n-2)!\,2!} = 66$

$\frac{n(n-1)(n-2)!}{(n-2)!} = 66 \times 2!$

$n^2 - n = 132$

$n^2 - n - 132 = 0$

$(n+11)(n-12) = 0$

$n = -11, n = 12 \quad \therefore n = 12$

6 Solve the equation $16(n-1)! = 5n! + (n+1)!$ where $n \in \mathbb{Z}^+$.

$16(n-1)! = 5n! + (n+1)!$

$16(n-1)! = 5n! + (n+1)n!$

$16(n-1)! = (n+6)n!$

$\frac{16(n-1)!}{(n-1)!} = (n+6)n$

$n^2 + 6n - 16 = 0$

$(n+8)(n-2) = 0$

$n = -8, n = 2$

$\therefore n = 2$

7 On a bookshelf there are four mathematics books, three science books, two geography books and four history books. The books are all different.

a In how many different ways can the books be arranged on the shelf?

b In how many ways can the books be arranged so that books of the same subject are grouped together?

(a) $(4+3+2+4)!$

$= 13!$

$= 6227020800$

(b) $4!\,(4! \times 3! \times 2! \times 4!)$

$= 165\,888$

8 A safe has two dials, one with 26 letters and one with the digits 0 to 9.

In order to open the safe, Rose has to choose a code consisting of three distinct letters followed by two distinct digits. Determine how many different safe codes are possible.

${}^{26}P_3 \times {}^{10}P_2$

$= 1404000$

9 A delegation of five students is to be selected for a Model United Nations conference. There are 10 boys and 13 girls to choose from.

a In how many different ways can a delegation be chosen if there are no restrictions?

b If the team is to include at least one girl and one boy, in how many ways can the delegation be selected?

(a) ${}^{23}C_5$

$= 33\,649$

(b) ${}^{23}C_5 - ({}^{13}C_5 + {}^{10}C_5)$

$= 32\,110$

10 a How many four-digit numbers can be made using the digits 0, 1, 3, 4, 5, 8 and 9?

b How many of the four-digit numbers have no repeated digits?

c How many four-digit even numbers can be made using the digits?

d How many of these even numbers are divisible by 5?

(a) $6 \times 7 \times 7 \times 7$

$= 2058$

(b) ${}^{6}P_1 \times {}^{6}P_3$

$= 720$

(c) $6 \times 7 \times 7 \times 3$

$= 882$

(d) $6 \times 7 \times 7 \times 1$

$= 294$

11 Graeme is training for a 10 km run. He has six different routes to choose for his training and he trains four times a week. He calculates that he will just manage to run a different set of routes each week leading up to his next race.

How many weeks are there before Graeme's race?

${}^{6}C_4 = 15$

12 A group of 12 people want to go to a concert. They can travel in a small car that takes one driver and one passenger and two cars each taking one driver and four passengers. If there are five drivers in the group, in how many different ways can they travel?

${}^{5}P_3 \times 9 \times {}^{8}C_4 \times {}^{4}C_4$

$= 37\,800$